**viscoelastic modeling of porcine ligaments**

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**Abstract.** Viscoelastic quasi-linear analytical models, as Fung, was implemented through the utilization of experimental results obtained from several porcine ligaments as: lateral collateral ligament (LCL), anterior cruciate ligament (ACL), posterior cruciate ligament (PCL) and medial collateral ligament (MCL). To implement quasi-linear viscoelastic models for soft tissues, as the Fung one, was necessary the utilization of a programming language, as C Sharp, and Object-oriented programming to deal with the model’s mathematical demands, as the convolution calculations. Moreover, those technologies allow to reduce the code execution time which was one of the main problems. Despite this benefit, was necessary to implement the numerical methods used in process. The models’ results show the stress evolution in relaxation tests. Although, the preliminary results show a good correlation between experimental and analytical models, showing a noticeable change in ligaments stiffness after the experimental implementation of relaxation tests.

**Keywords:** knee ligaments, analytic model, viscoelasticity, Fung

1. Introduction

The knee is one of the most complex joints in the body being subjected to different efforts. Studying and correctly describing the mechanical behavior of the knee ligaments is extremely important. Thus, a huge number of researches were published trying to macroscopically analyze these tendons through different viscoelastic mechanical models. (Rossetto, 2009) shows that this knowledge is important for better analyzes to be made to determine physical training, such as in cases of therapy for tendinopathies. (Bernardes et. al, 2005) sought to determine the biomechanical parameters for modeling the human knee joint through extensive exercises, together with images obtained by videofluoroscope, where viscoelasticity plays an important role.

Viscoelasticity is understood as the property of materials that present viscous and elastic behavior at the same time, being a concept widely used in various sectors of the industry. The simplest viscoelastic model is one that considers linear functions, where the creep compliance and stress relaxation functions depending only on time, it is commonly used for metals. (Tareco, 2014) uses Maxwell and Kelvin linear models to model a steel-concrete structure, analyzing the relaxation and creep compliance just for the concrete in the mixed structure response. Moreover, as presented by (Queiroz, 2008), viscoelastic materials are also used to attenuate vibrations and noise in structures, having application in both the automotive and aerospace sectors.

The quasi-linear viscoelastic model, proposed by (Fung, 1993), is commonly used in soft tissue research since it describes behavior close to reality and, like any model, it has limitations, since different sets of constants are found for different relaxations. (Piazza et al., 2001) developed a three-dimensional dynamic model of the tibiofemoral and patellofemoral articulations to predict the knee implant movements during a step-up activity. They were based on the Fung’s model, using dynamic equations of motion subjected to forces generated by muscles, ligaments, and contact at articulations, and achieved good results for the flexion-extension angle of the knee, but not for translations at the tibiofemoral articulations. (Debski et al, 2004) applied the Fung’s model and analyzed the viscoelastic properties of the healing goat medial collateral ligament, MCL. For that, they characterized the reduced relaxation function and the elastic response and demonstrated that que quasi-linear viscoelastic model could be successfully used to describe the MCL viscoelastic behavior during the healing phases.

Moreover, the quasi-linear viscoelastic method is frequently applied with computational resources since it has complex equations that do not have analytical results. (Xu and Engquist, 2018) proposed a mathematical model for relaxation modulus based on nonlinear model and its numerical solution and developed a finite-element framework and a numerical algorithm to implement this model for simulating responses under static and dynamic loadings. They validated the model using many materials comparing both experimental and numerical results. (Weiss et al, 2001) reviewed past and current techniques for the computational modeling of soft tissues, showing relevant concepts from fields of continuum mechanics and finite element and emphasizing the microstructural influence of soft tissues. (Abramowitch et al., 2004) obtained the constants for quasi-linear viscoelastic model that are used to describe the elastic response (constants A and B) and the reduced relaxation function (constants C, and ) with an improved approach that converges to a single solution with minimal variation and subjected six goat femur-medial collateral ligament-tibia to a uniaxial tensions test considering ramp time. In tests, the convergence failed for three ligaments, with the biggest errors at constants A, B e .

Through the above, the aim of this paper is to explain how to implement numerically the Fung’s quasi-linear viscoelasticity model, showing the flowcharts and the results. Using the C# programming language (Wagner et al, 2021) and ASP.NET MVC framework (Rick Anderson, 2019) (Gasparotto, 2014), a REST API was developed (Silveira, 2020) capable of performing the necessary calculations for this model and generating a CSV file to compare the numerical results with the experimental ones. The API was developed focusing on scalability, maintainability, and readability, applying some design patterns, as Strategy, and object-oriented programming patterns, as SOLID principles, and some resources were also used to optimize that software, as Swagger, for building the user interface. Finally, both numerical and experimental results will be compared for each ligament.

1. Fung’s quasi-linear viscoelastic model

The quasi-linear viscoelastic model, proposed by (Fung, 1993), applies the non-linearity stress-strain relation expressing the stress in two parts: the reduced relaxation function, which depends only on time, and elastic response, which depends on strain. To improve the stress calculations, the elastic response can be expressed depending only on time, because, in that research, the strain is considered depending on this, as will be shown below. This model is commonly used for soft tissue, as it can represent the tissue with good approximation. The constants needed for the equations is obtained experimentally, however, like any model, the Fung’s model has limitations, since for distinct relaxations and strain levels, different values for those constants are found. Moreover, two considerations were made: consider and disregard ramp time. While considering, is possible to calculate the variables A and B, shown below in elastic response equation, with those, multiple relaxations can be assumed. In that research, is assumed just two relaxations. In table 1, the Fung’s model constants for each tissue are presented with their value for the first and second relaxation, these were obtained by the research team experimentally, as mentioned above.

Table 1. Fung’s model constant for first and second relaxation used for each tissue.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Material Constants** | **ACL** | | **PCL** | | **LCL** | | **MCL** | |
| **First** | **Second** | **First** | **Second** | **First** | **Second** | **First** | **Second** |
| **G1** | 0.27 | 0.063 | 0.03 | 0.028 | 0.10 | 0.11 | 0.018 | 0.02 |
| **G2** | 0.18 | 0.098 | 0.05 | 0.046 | 0.18 | 0.17 | 0.032 | 0.03 |
| **G3** | 0.56 | 0.24 | 0.11 | 0.077 | 0.32 | 0.24 | 0.059 | 0.04 |
| **G∞** | 1.81 | 1.64 | 1.43 | 1.387 | 3.79 | 3.55 | 0.86 | 0.81 |
| **τ1**, s | 19.33 | 2.35 | 0.97 | 1.67 | 1.02 | 1.88 | 0.59 | 0.96 |
| **τ2**, s | 19.6 | 19.86 | 6.92 | 13.91 | 7.52 | 15.63 | 4.73 | 9.38 |
| **τ3**, s | 370.8 | 213.64 | 53.18 | 134.91 | 53.38 | 150.0 | 32.29 | 95.36 |
| **A, MPa** | 112.96 | 54.88 | 1.86 | 1.77 | 3.94 | 7.27 | 9.47 | 11.58 |
| **B** | 0.34 | 0.86 | 10.15 | 12.79 | 1.19 | 6.17 | 1.39 | 1.77 |

* 1. Mathematical equation

(Fung, 1993) propose equations for elastic response, reduced relaxation function and stress considering one relaxation, although, as two relaxations will be considered, it is necessary to reformulate these equations. Moreover, each parameter will be expressed differently when considering and disregarding ramp time, except for reduced relaxation function.

* + 1. Strain

The equations used to describe the strain were developed to represent the experiments. When considering ramp time, the strain behavior is expressed in equation 1, that when the strain maintains at the maximum value , it represents the relaxations and when stays at the minimum value , represents the recovery, that behavior also is observed in (Duenwald, et al., 2009). When disregard ramp time, the equation 2 is used, where is considered a constant strain while all experiment.

|  |  |
| --- | --- |
| , | (1) |

where, the parameters and represent, respectively, ramp time and strain rate applied in experiment, with used when strain increase and , when decrease. Furthermore, the parameters , , and are the limit time for each equation, indicating when the strain behavior changes.

|  |  |
| --- | --- |
| , | (2) |

where, represents the constant stain applied in experiment.

With that, is possible to calculate the derivative that will be used in the stress calculations step. The equation 3 and 4 are the derivative in time of, respectively, equations 1 and 2.

|  |  |
| --- | --- |
| , | (3) |
| . | (4) |

* + 1. Elastic response

The elastic response corresponds the soft tissue elastic part. As mentioned previously, two equations will be used to describe the elastic response. When considering ramp time, an exponential approximation has been chosen like used in research (Abramowitch, 2004).

|  |  |
| --- | --- |
| , | (5) |

where constants A, in Pa (Pascal), and B, dimensionless, are material constants and represents, respectively, elastic stress constant and elastic power constant. Moreover, as shown previously, the equation 5 is rewritten with the aim to elastic response only depends in time.

|  |  |
| --- | --- |
| , | (6) |

When disregarding ramp time, the elastic response is considered constant for all time domain, because the strain is constant, and that parameter depends on strain.

|  |  |
| --- | --- |
| , | (7) |

where, is the initial stress applied in experiment.

As made for strain, the derivative for elastic response must be calculated because it will be used in equations for describe the stress.

The derivative in time and in strain for equation 6:

|  |  |
| --- | --- |
| , | (8) |
| . | (9) |
| The derivative in time and in strain for equation 7:  . | (10) |

* + 1. Reduced relaxation funcion

The reduced relaxation function represents the viscous portion and occurs for all time domain begging at 1, **g**(0) = 1. According with (Fung, 1993), it can be described in two ways. The first, equation 9, also called the simplified reduced relaxation function, is written as the Prony Series, but using only three elements in the sum, while according with (Babaei et al, 2015), those are sufficient for a good approximation, and (Funk et al., 2000) more than three elements do not result in significant gain. The second, equation 10, was developed from Kelvin model, standard linear solid (Fung, 1993), and uses integrals that only have numerical solutions. Moreover, both equations were implemented and tested but only the first was used, as constants are easier to be calculated experimentally.

|  |  |
| --- | --- |
| , | (9) |

where and are material dimensionless constants called relaxation modulus and represents the amplitude of the stress curve in relaxation, and is the relaxation time in seconds, also a material constant.

|  |  |
| --- | --- |
| , | (10) |

where C, and are material constants and represents, respectively, a dimensionless relaxation constant, fast and slow relaxation times in second. To improve the numerical implementation, that equation was rewritten as shown below.

Based on material properties and the constants definition, can be assumed that , so, , therefore, could be rewritten like:

|  |  |
| --- | --- |
| , |  |

Then:

|  |  |
| --- | --- |
| , | (11) |

Applying equation 11 in 10, is found:

|  |  |
| --- | --- |
| , | (12) |

Also calculating the derivative in time for each equation for reduced relaxation function.

Deriving (9):

|  |  |
| --- | --- |
| , | (13) |

Deriving (12):

|  |  |
| --- | --- |
| , | (14) |
| , |  |

Applying the definition of calculus to the derivative of a definite integral:

|  |  |
| --- | --- |
| , |  |

where , and b.

|  |  |
| --- | --- |
| , | (15) |

Applying (15) in (14):

|  |  |
| --- | --- |
| , | (16) |

* + 1. Stress

(Fung, 1993) shows three equivalent equations to calculate the stress:

|  |  |
| --- | --- |
| , | (17) |
| , | (18) |
| , | (19) |

As mentioned above, the elastic response and reduced relaxation function can be expressed only depending on time, so the partial derivative can be changed by total derivative. Moreover, and .

|  |  |
| --- | --- |
| , | (20) |
| , | (21) |
| , | (22) |

While considering ramp time, equations 20 and 22 return satisfactory results, however the results obtained with equation 21 diverged from the others, like shown in Fig. 8. Disregarding ramp time, the elastic response is constant, and its derivative is zero for all time domain, as shown previously. Thus, the equation 20 cannot be used, because it always returns zero since, and equations 21 and 22 can be rewritten.

Rewriting (21):

|  |  |
| --- | --- |
| , |  |
| , |  |
| , |  |
| , |  |
| . | (23) |

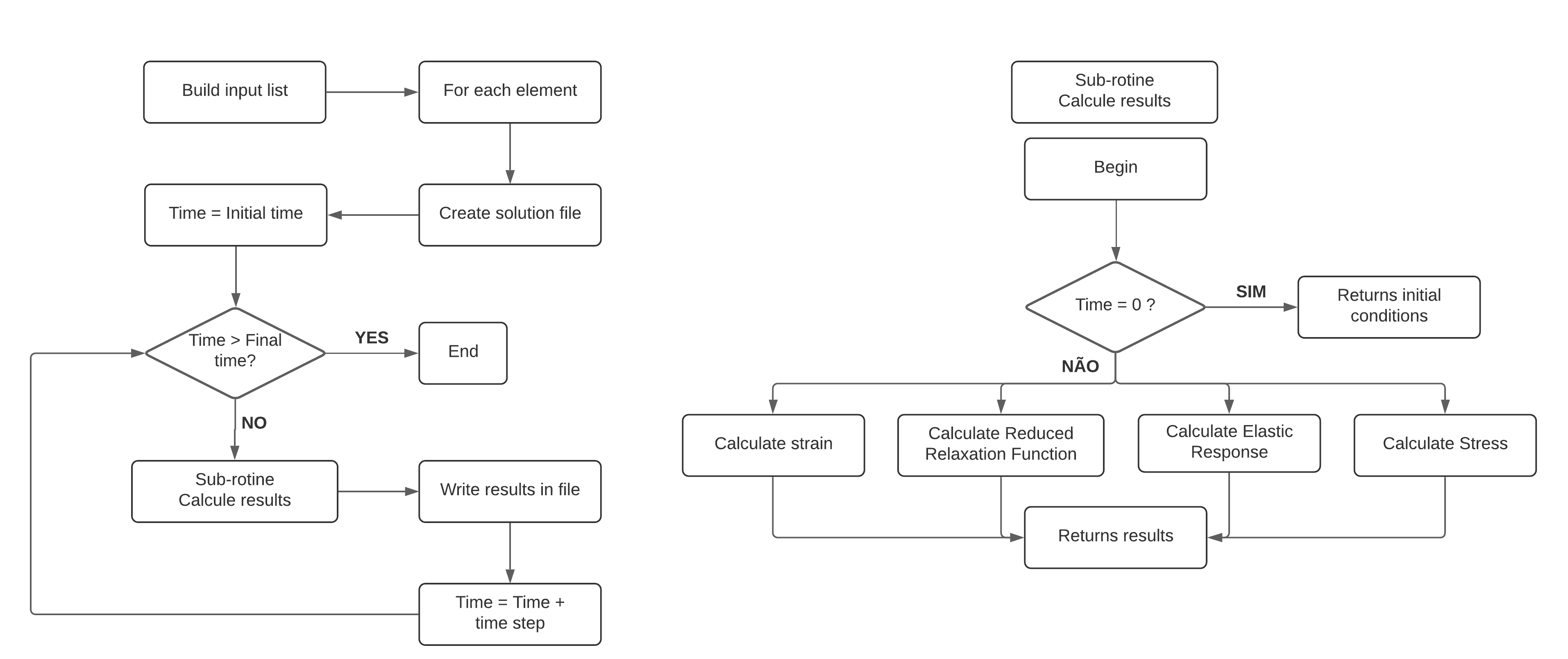
Rewriting (22):

|  |  |
| --- | --- |
| , |  |
| , |  |
| . | (24) |

As presented, (23) and (24) are equals, so when disregarding ramp time, a unique equation can be used.

1. numerical implementation

The numerical implementation of Fung’s Model was developed by two steps: creating a class that represents the model and contains the equations for each parameter and its derivative in time; and creating a class to orchestrate the operation. Also was created an artificial frontier in the code that separate the operation and the model, being created specific contracts for each one. This was done based on Single-Responsibility Principle that gets easier to implement resources, prevents unexpected side-effects and improves maintainability. It is noteworthy that the execution time was minutes and, in worst cases, hours, because the equations used to calculate the stress are not optimized for numeric applications. To improve that was used the class Task, a native resource from C#, with the aim to let some steps asynchronous, executing multiple tasks together and reducing the execution time. It was used in both classes mentioned, in first, when calculating the results, and, in second, when iterating the input list, reducing that time to seconds, in worst case.



(a) (b)

Figure 1. Flowchart for (a) main operation and (b) sub-routine “Calculate Results”.

The class that represents the model also contains a method, represented in Fig. 1 as sub-routine “Calculate Results”, that calculate in parallel all results necessaries - strain, elastic response, reduced relaxation function and stress – as shown in Fig. 1.b, and returns those values in an object. The orchestrator, as called, is responsible to orchestrate the operation, executing each step shown on Fig. 1.a, furthermore, previously the request data is validated to certain if it is valid and any error will be thrown during code execution.

What is more, it was necessary to implement numerical methods to deal with integrations and derivatives present in stress and reduced relaxation function equations. For the integrals, the Composite Simpson's Rule, equation 25, (Regra de Simpson, 2021) (Regras Compostas, 2021) to grant greater precision, since this was the best solution found when compared with other methods using a same time step. For the derivatives, the Symmetric Derivative, equation 26, (Da Cruz, 2012) was used since it gives the precision necessary while calculating the parameters.

|  |  |
| --- | --- |
| , | (25) |

where f(x) is an integrable function, a and b are the limits of integration, x is a differential of the variable x, and N is the number of subdivisions.

|  |  |
| --- | --- |
| , | (26) |

where f(x) is a differentiable function and x is a differential of the variable x.

1. Numerical extrapolation

Just implement numerically the quasi-linear viscoelastic model is not enough, for a better comparison between experimental and numeric, it was necessary to develop a routine for extrapolating the experimental results. For this, it was necessary to predict the next values ​​based on the stress curves' behavior. Analyzing this, it is possible to notice two important behaviors during relaxation: the stress decreases on time and the concavity is upwards, so this indicates that the stress first and second derivative on time are, respectively, positive and negative. Hereupon, this logic must be used while validating each point before extrapolation to remove invalid points that may interfere while extrapolating the results.

Diagrama

Descrição gerada automaticamente

Figure 2. Flowchart for numerical extrapolation.

The numerical extrapolation was made according to flowchart at Fig. 2, it is noteworthy that after the API receives input data, these are validated to ensure that the file has enough lines for the operation and the correct parameters were passed. The operation was divided in two subroutines to improve maintainability and readability, since the software may be used in future researches.

1. RESULTS AND CONCLUSIONS

In that study, it was possible to describe a behavior similar to reality, even with some divergences, the shape of the curves obtained numerically reproduced the experimental. The best results were found for the case when disregarding the ramp time, since the equation used is very simple.

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